Financial Mathematics: Continuous Stochastic Models (C001814)

Course size *(nominal values; actual values may depend on programme)*

Credits 6.0   **Study time 165 h**

Course offerings in academic year 2024-2025

Lecturers in academic year 2024-2025

Vanmaele, Michèle	WE02	lecturer-in-charge

Offered in the following programmes in 2024-2025	**crdts**	**offering**

Teaching languages

Dutch

Keywords

Stochastic processes, martingales, arbitrage, Black-Scholes, financial derivatives, (in)complete markets, hedging, interest rate models

Position of the course

In this course the theory of financial derivatives such as options is developed in a continuous setting. This course is a continuation of the course Financial mathematics: discrete stochastic models.
The aim of the course is to derive the continuous one- and multidimensional Black-Scholes model for option pricing and to study hedging strategies for it. Special attention is paid to more complex financial derivatives such as Asian options and other exotic options. This course will also deal with the study of interest rate models and pricing techniques of interest rate derivatives.

Contents

1. Continuous stochastic processes:
• Filtrations, conditional expectation
• Brownian motion and quadratic variation
• Markov property
• Stochastic integral, Itô-calculus
• One- and multi-dimensional Itô-formula
• Stochastic differential equation and relation to partial differential equations
2. Financial market models
• Equivalent martingale measures
• Risk-neutral pricing
• Martingale representation
• Change of numeraire
3. One- and multidimensional Black-Scholes model
• Pricing of options, forwards and futures
• Static and dynamic hedgen, the Greeks
• Volatility
4. Interest rate models

Initial competences

Basic knowledge of probability and statistics as taught in the Bachelor of Mathematics.
Preferably knowledge of financial algebra as taught in the course "Financial mathematics" in the minor Economics and of discrete pricing models for financial derivatives as taught in the course "Financial mathematics: discrete stochastic

models".

Final competences

1 Analyse, discuss and apply probabilistic concepts and properties from financial mathematics.

2 Apply Itō-calculus for martingales, solve stochastic differential equations by means of the Feynmac-Kac representation, be able to understand the extension of the Itō-calculus to semi-martingales.

3 Expose knowledge of the theory of risk-neutral pricing (equivalent martingales, martingale representation, change of numeraire) and apply this theory.

4 Explain the fundamental concepts in financial mathematics (arbitrage, complete market, ...) and apply them to the Black-Scholes model.

5 Build and discuss the Black-Scholes model, identify, discuss and develop different hedging strategies; in particular pricing of options when the underlying asset pays a dividend.

6 Identify, discuss and price studied as well as new financial derivatives.

7 Distinguish between a complete and an incomplete market.

8 Describe and discuss different interest rate models, derive from those models the distribution of the instantaneous interest rate and price the corresponding bonds and options.

9 Link a new problem to a studied problem and apply the techniques which were there developed.

Conditions for credit contract

Access to this course unit via a credit contract is determined after successful competences assessment

Conditions for exam contract

This course unit cannot be taken via an exam contract

Teaching methods

Seminar, Lecture

Study material

Type: Handbook

Name: Stochastic Calculus for Finance II: Continuous-Time Models
Indicative price: € 60
Optional: yes
Language : English
Author : Steven E. Shreve
ISBN : 978-0-38740-101-0
Online Available : Yes
Available in the Library : Yes

Type: Slides

Name: Stochastic Calculus for Finance II: Continuous-Time Models
Indicative price: Free or paid by faculty
Optional: no
Language : Dutch
Available on Ufora : Yes
Online Available : Yes

References

1 T. Björk, Arbitrage Theory in Continuous Time, Oxford University Press, 1998

2 D. Brigo & F. Mercurio, Interest Rate Models: Theory and Practice, 2001

3 R.A. Dana & M. Jeanblanc, Financial Markets in Continuous Time, Springer-Verlag, New York, 2003

4 J.C. Hull, Options, Futures, and other Derivatives, Prentice Hall, 4th edition, 2000

5 D. Lamberton & B. Lapeyre, Introduction to Stochastic Calculus Applied to Finance, Chapman & Hall, Londen, 1996

6 T. Mikosch, Elementary Stochastic Calculus with Finance in View, in: Advanced Series in Statistical Science & Applied Probability, Vol. 6, editor: O. E. Barndorff-Nielsen, World Scientific, Singapore, 1998

7 L.C.G. Rogers, Stochastic Calculus and Markov Methods, in: Mathematics of Derivative Securities, editors: M.A.H. Dempster en S.R. Pliska, Cambridge University Press, Cambridge, 1997

8 S.E. Shreve, Stochastic Calculus for Finance II: Continuous-Time Models,

Springer-Verlag, New York, 2004

9 P. Wilmott, J. Dewynne & S. Howison, Option Pricing, Mathematical models and computation, Oxford Financial Press, Oxford, 1993

Course content-related study coaching

During lectures and exercise sessions the students are welcome to ask questions. The lecturer is also available for supplementary help. There is interactive support through Ufora (forum, email, links).

Assessment moments

end-of-term assessment

Examination methods in case of periodic assessment during the first examination period

Written assessment with open-ended questions

Examination methods in case of periodic assessment during the second examination period

Written assessment with open-ended questions

Examination methods in case of permanent assessment

Possibilities of retake in case of permanent assessment

not applicable

Extra information on the examination methods

Written examen, a formularium may be used.

Calculation of the examination mark

Written examen: 100%