

## Banach Spaces and Banach Algebras (C003012)

**Course size** *(nominal values; actual values may depend on programme)*

**Credits 6.0**

**Study time 165 h**

**Course offerings and teaching methods in academic year 2023-2024**

A (semester 1)	Dutch	Gent	lecture seminar
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**Lecturers in academic year 2023-2024**

Vernaeve, Hans	WE16	lecturer-in-charge
Kumar, Vishvesh	WE16	co-lecturer
Rottensteiner, David	WE16	co-lecturer

**Offered in the following programmes in 2023-2024**

	crdts	offering
<a href="#">Master of Science in Teaching in Science and Technology(main subject Mathematics)</a>	6	A
<a href="#">Master of Science in Mathematics</a>	6	A

**Teaching languages**

Dutch

**Keywords**

Banach spaces, Banach algebras,  $C^*$ -algebras

**Position of the course**

In this course, abstract analysis is developed more deeply. In the bachelor courses, metric spaces and Hilbert spaces were covered. Banach spaces were treated to a less extent. A lot of vector spaces of (generalized) functions, linear operators, ... in analysis are Banach spaces which aren't Hilbert spaces. Moreover, they often are Banach algebras (and even  $C^*$ -algebras) for multiplication. The course also aims to provide a preparation for research on (generalized) functions and operators and to generalizations in the theory of topological vector spaces (course Capita selecta in analysis).

**Contents**

1. Banach spaces, linear operators on Banach spaces, dual spaces, weak topology, Hahn-Banach and Banach-Alaoglu theorem, reflexivity
2. Representation of separable normed spaces as spaces of continuous mappings
3. Baire and Banach-Steinhaus theorems, Open mapping and closed graph theorems
4. Banach algebras, spectrum, Gelfand-Mazur theorem
5. Commutative Banach algebras and  $C^*$ -algebras, Banach algebras of continuous maps on a topological space, Gelfand-Naimark representation theorem, Stone-Weierstrass theorem. Application: Wiener's Tauberian theorem
6. non-commutative  $C^*$ -algebras, functional calculus, algebras of boundend linear operators on a Hilbert space, spectral theorems.
7. Optional topics: Stone-Cech-compactification, GNS-construction, introduction to topological vector spaces, regular Banach algebras.

**Initial competences**

The student has successfully completed the course "Topology and metric spaces" or has acquired the final competences of this course in another way.  
Familiarity with analysis in one complex variable and Hilbert spaces is also assumed.

**Final competences**

- 1 The student has insight in how the abstract Banach space and Banach algebra theory relates to a number of abstract concepts in analysis from the courses mentioned in the initial competences.

- 2 The student understands how Banach space and Banach algebra theory is useful in concrete situations in analysis (such as algebras of functions or linear operators).
- 3 The student understands how algebra and analysis contribute to a theory which would have a much more technical character when only one of the building blocks were used.
- 4 The student understands how a number of concepts in Banach space theory are developed as tools to compensate the lack of local compactness in finite dimensional vector spaces.
- 5 The student understands how a number of concepts in Banach space theory are developed as tools to compensate the lack of an inner product in Hilbert spaces.

#### **Conditions for credit contract**

Access to this course unit via a credit contract is determined after successful competences assessment

#### **Conditions for exam contract**

This course unit cannot be taken via an exam contract

#### **Teaching methods**

Seminar, Lecture

#### **Extra information on the teaching methods**

Due to the corona crisis, modified teaching methods can be installed if necessary.

#### **Learning materials and price**

Lecture notes, available electronically on the Ufora platform.

#### **References**

R. Kadison and J. Ringrose, Fundamentals of the theory of Operator Algebras, vol. 1, Elementary theory, Academic Press, 1983.  
W. Rudin, Functional Analysis, McGraw-Hill, 1973.

#### **Course content-related study coaching**

The lecturer is available for answering individual questions, also outside of the lecture periods (on appointment). Students can also have their independently solved exercises corrected.

#### **Assessment moments**

end-of-term assessment

#### **Examination methods in case of periodic assessment during the first examination period**

Oral assessment, Written assessment

#### **Examination methods in case of periodic assessment during the second examination period**

Oral assessment, Written assessment

#### **Examination methods in case of permanent assessment**

#### **Possibilities of retake in case of permanent assessment**

examination during the second examination period is possible

#### **Extra information on the examination methods**

Exercises: open book examination

#### **Calculation of the examination mark**

100% periodic evaluation.