GHENT UNIVERSITY

Course Specifications

Valid as from the academic year 2024-2025

Linear Algebraic Groups (C003013)

Course size *(nominal values; actual values may depend on programme)*

Credits 6.0 **Study time 165 h**

Course offerings and teaching methods in academic year 2024-2025

A (semester 2)	English	Gent	seminar
			lecture

Lecturers in academic year 2024-2025

De Medts, Tom	WE01	lecturer-in-charge

Offered in the following programmes in 2024-2025	**crdts**	**offering**
Master of Science in Teaching in Science and Technology(main subject Mathematics)	6	A
Master of Science in Mathematics	6	A
Exchange Programme in Mathematics (master's level)	6	A

Teaching languages

English

Keywords

Linear algebraic groups, Hopf algebras, classical groups, algebraic geometry, Lie algebras, diagonalizability, unipotent groups, tori, reductive groups, root systems.

Position of the course

The study of important algebraic structures, the knowledge of which is essential in mathematical research in algebra and related topics, and of the basics of algebraic geometry. The theory is presented in a sufficiently general and abstract context to help the student reach a higher level of abstraction.

Contents

The central topic of this course is the study of linear algebraic groups. After a short introductory overview, we begin with the necessary setup of the mathematical tools that we will use, in particular algebras, tensor products, a short introduction to category theory (including the Yoneda Lemma) and a brief introduction to affine varieties.
We then introduce affine algebraic groups, in a functorial fashion. We study the corresponding coordinate algebra (Hopf algebras). We have a closer look at representations of linear algebraic groups and the corresponding comodule structure, and we show that every affine algebraic group is linear.
We continue with the Jordan decomposition of linear algebraic groups, and we have a closer look at diagonalizibility and triagonalizability (including the Lie-Kolchin theorem).
We discuss the Lie algebra of a linear algebraic group, and we further develop some important topological aspects, such as connectedness and smoothness.
We study the Weyl group and the root system associated to a reductive linear algebraic group over an algebraically closed field, and we illustrate this with examples. This culminates in the classification of semisimple linear algebraic groups over algebraically closed fields, but we do not prove all the details rigorously in this process.

Initial competences

Knowledge of linear algebra (vector spaces over arbitrary fields), and basic concepts in algebra (groups, rings, ideals,...).

Final competences

1 The student is familiar with the essential concepts from the theory of linear algebraic groups. He/she is skilled in the modern functorial approach to the theory.
2 The student is able to apply the introduced techniques and to construct abstract algebraic

arguments on his own.

3 The student is able to follow arguments where some parts are not proven explicitly. He/she learns how to think intuitively before delving into the details of a proof.

Conditions for credit contract

Access to this course unit via a credit contract is determined after successful competences assessment

Conditions for exam contract

This course unit cannot be taken via an exam contract

Teaching methods

Seminar, Lecture

Study material

Type: Syllabus

Name: Linear Algebraic Groups
Indicative price: Free or paid by faculty
Optional: no
Language : English
Number of Pages : 147
Available on Ufora : Yes
Online Available : Yes
Available in the Library : No
Available through Student Association : No

References

- P. Abramenko, Lineare algebraische Gruppen, Eine elementare Einführung, Johann Wolfgang Goethe-Universität, Frankfurt am Main, 1990.
- A. Borel, Linear Algebraic Groups, Second enlarged edition, Graduate Texts in Mathematics 126, Springer, 1991.
- J. E. Humphreys, Linear Algebraic Groups, Springer-Verlag, New York, Heidelberg, Berlin, 1975.
- K. McGerty, Algebraic Groups and Representation Theory. Available online: http://www2.imperial.ac.uk/~kmcgerty/teaching.html.
- J.S. Milne, Basic Theory of Affine Group Schemes; Lie Algebras, Algebraic Groups, and Lie Groups; Reductive Groups. Available online: http://www.jmilne.org/math/CourseNotes/ala.html.
- T. A. Springer, Linear Algebraic Groups, Second edition, Progress in Mathematics vol. 9, Birkhäuser, 1998.
- T. Szamuely, Lectures on Linear Algebraic Groups. Available online: http://www.renyi.hu/~szamuely/lectures.html.

Course content-related study coaching

During the interactive lectures definitions and techniques are explained and guidelines on how and what to prepare for the lessons will be given. The student prepares the exercises and in exercise sessions these are worked out and discussed depending on the possible questions and/or problems encountered by the students. Of course, there is also the possibility to consult the lecturer for further questions.

Assessment moments

end-of-term assessment

Examination methods in case of periodic assessment during the first examination period

Oral assessment, Written assessment with open-ended questions

Examination methods in case of periodic assessment during the second examination period

Oral assessment, Written assessment with open-ended questions

Examination methods in case of permanent assessment

Possibilities of retake in case of permanent assessment

not applicable

Extra information on the examination methods

Periodic evaluation for the theory and the exercises, where the students should show in how far they have mastered the introduced concepts and abstract arguments. For both parts, the

student is allowed to use all course material; thus the emphasis is on the understanding of the material, and not on the ability to reproduce it.

Calculation of the examination mark

- Periodic evaluation (50%) for the theory
- periodic evaluation (50%) for the exercises