GHENT UNIVERSITY

Course Specifications

Valid as from the academic year 2024-2025

Analytic Number Theory (C003824)

Course size *(nominal values; actual values may depend on programme)*

Credits 6.0 **Study time 165 h**

Course offerings and teaching methods in academic year 2025-2026

A (semester 2)	Dutch, English	Gent	seminar	15.0h
			lecture	30.0h

Lecturers in academic year 2025-2026

Debruyne, Gregory	WE16	lecturer-in-charge
Chen, Bin	WE16	co-lecturer

Offered in the following programmes in 2025-2026	**crdts**	**offering**
Master of Science in Teaching in Science and Technology(main subject Mathematics)	6	A
Master of Science in Mathematics	6	A
Exchange Programme in Mathematics (master's level)	6	A

Teaching languages

English, Dutch

Keywords

Number theory, divisibility, arithmetic functions, Riemann zeta function, the prime number theorem, Dirichlet series, Dirichlet L-series, prime numbers in arithmetic progressions, Tauberian theorems.

Position of the course

The study of properties of the integers and the use of a number of techniques from mathematical analysis. Making clear why certain number theoretic problems, although formulated in simple terms, should be approached with some advanced methods. The student learns fundemental analytic tools and how they can be applied to concrete questions in number theory.

Contents

1 Divisibility: greatest common divisor, prime numbers (how many primes are there and how are they distributed?).
2 Arithmetic functions: Dirichlet convolution, asymptotic formulas and estimates, Euler summation formula, applications of summation by parts, averages of arithmetic functions, the Dirichlet hyperbola method.
3 Elementary results on the distribution of prime numbers: Chebyshev estimates, Mertens estimates, prime theorem elementary equivalences.
4 Dirichlet series: algebraic and analytic properties of Dirichlet series, Euler products, the Riemann zeta function, Mellin transform, Perron inversion formulas.
5 The prime number theorem and Tauberian theorems: a short way to the prime number theorem via contour integration, complex Tauberian theorems
6 The error term in the prime number theorem and properties of the Riemann zeta function: growth estimates for the Riemann zeta function and its reciprocal, zeros of the Riemann zeta function, the prime number theorem with remainder.
7 Prime numbers in arithmetic progressions: Dirichlet theorem, Dirichlet characters, Dirichlet L-series.

Initial competences

Basic knowledge of number theory. Knowledge of classical and complex analysis. Insights in the notions of groups and rings.

Final competences

1 Some insight in the characteristic problems from number theory.
2 Knowing some proofs and methods.
3 Working knowledge with arithmetic functions and Dirichlet series calculus and its use in applications.
4 Being able to apply methods from analysis in number theory.

Conditions for credit contract

Access to this course unit via a credit contract is determined after successful competences assessment

Conditions for exam contract

This course unit cannot be taken via an exam contract

Teaching methods

Seminar, Lecture

Extra information on the teaching methods

Theory: lectures and interactive seminar in which examples will be worked out.
Exercises: the student prepares the exercises in advance and then these are worked out during the exercise sessions. Some proofs and applications will be studied in the exercises.
Learning material: Syllabus, price (approx.): EUR 10.00.

Study material

Type: Syllabus

Name: Introduction to Analytic Number Theory. Lecture Notes by A. J. Hildebrand, University of Illinois at Urbana-Champaign, 2013
Indicative price: € 10
Optional: no
Language : English
Number of Pages : 197
Oldest Usable Edition : 2013 Edition
Available on Ufora : Yes
Online Available : No
Available in the Library : No
Available through Student Association : No

References

Apostol, T. M., Introduction to analytic number theory, Springer-Verlag, 1976.
Bateman, P. T., Diamond, H. G., Analytic number theory. An introductory course, World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, 2004.
Tenenbaum, G., Introduction to analytic and probabilistic number theory, Cambridge University Press, Cambridge, 1995.
Vinogradov, I. M., An introduction to the theory of numbers, Pergamon Press, 1955.

Course content-related study coaching

Coaching during the exercise sessions. Availability for questions. Didactic material via Ufora.

Assessment moments

end-of-term assessment

Examination methods in case of periodic assessment during the first examination period

Written assessment with open-ended questions

Examination methods in case of periodic assessment during the second examination period

Written assessment with open-ended questions

Examination methods in case of permanent assessment

Possibilities of retake in case of permanent assessment

not applicable

Extra information on the examination methods

Written exam. The exam questions are meant to test the student knowledge on the new concepts and techniques and their relation with the theory. They evaluate the

students insights in the theory, but emphasis is given to the application of the techniques in concrete situations.

Calculation of the examination mark

Periodic evaluation (100%)