GHENT UNIVERSITY

Course Specifications

Valid as from the academic year 2025-2026

Advanced Numerical Methods (C004011)

Course size *(nominal values; actual values may depend on programme)*

Credits 6.0 **Study time 180 h**

Course offerings and teaching methods in academic year 2025-2026

A (semester 2)	English	Gent	lecture
			seminar

Lecturers in academic year 2025-2026

Köllermeier, Julian	WE02	lecturer-in-charge

Offered in the following programmes in 2025-2026	crdts	offering
Master of Science in Teaching in Science and Technology(main subject Mathematics)	6	A
Master of Science in Computer Science	6	A
Master of Science in Mathematics	6	A

Teaching languages

English

Keywords

Model reduction, multiscale numerical methods, hyperbolic PDEs, free-surface flow, gas dynamics

Position of the course

First half:
Physical phenomena in atmospheric re-entry, nuclear fusion or geophysical flows require tailored mathematical models to achieve accurate, yet feasible numerical simulations. This course will give an introduction to the process of model reduction via moment methods and then discuss properties and numerical solution of the obtained models.
We will introduce, among others, PDE models from rarefied gases and shallow water flows and investigate selected analytical properties of the models, e.g., hyperbolicity, equilibrium points, linear stability. Time permitting, we will discuss alternative model reduction techniques, such as low-rank approximations and dynamical low-rank approximations and applications of moment models in Uncertainty Quantification.
This part prepares students for research projects in the broad field of applied mathematics that include complex models for which the derivation of a reduced model is beneficial. Such applications can be in aerospace engineering, free-surface flows, nuclear fusion, radiotherapy, uncertainty quantification.

Second half:
Many physical processes from fluid dynamics are modeled by large hyperbolic PDE systems evolving on multiple scales, e.g., moment models for rarefied gases (atmospheric entry) and shallow water flows (tsunamis). These models require special numerical methods that are able to preserve the properties of the models during the numerical computation, while being accurate and stable. Additionally, many models contain stiff source terms in certain regimes, e.g., for large friction. In this part, we discuss specific numerical methods for a wide range of hyperbolic PDE models. We set the stage with (non-conservative) finite volume methods and continue with methods tailored to stiff systems like Projective Integration or micro-Macro decomposition. These methods are specifically designed for hyperbolic

models from fluid dynamics. In the end, we introduce adaptive time stepping from a control theory theory viewpoint to accelerate simulations evolving on different scales in time.
The target group are master students that are interested in applied mathematics, numerical simulation, fluid dynamics, or control theory.

Contents

part I:
1. Motivation
2. Introduction to kinetic theory
3. The method of moments
4. Nonlinear moment framework
5. Nonlinear moment equations for rarefied gases
6. Hyperbolic moment equations for rarefied gases
7. Filtered moment models
8. Hilbert expansion-based model reduction
9. Moment equations for shallow flows
10. Moment models for Uncertainty Quantification
11. Other model reduction techniques

part II:
1 Examples for Multi-scale systems
2 Finite Volume schemes
3 Standard methods for stiff models
4 Multi-scale methods for stiff models
5 Structure-preserving methods
6 Adaptive Time stepping

Initial competences

Assumed knowledge: differential equations numerical methods, e.g., as in Numerical Analysis, Mathematical Modelling

Final competences

At the end of the course, the student is able to:
1. apply different model reduction techniques to derive macroscopic PDEs from an underlying micro-/mesoscopic PDE description.
2. analyze properties like stability and hyperbolicity of the models.
3. use reduced models to solve application problems, e.g. from the fields of rarefied gases or free-surface flows.
4. choose from a variety of tailored methods to solve multi-scale hyperbolic PDEs.
5. understand the properties, advantages and disadvantages of the discussed numerical schemes.

Conditions for credit contract

Access to this course unit via a credit contract is determined after successful competences assessment

Conditions for exam contract

This course unit cannot be taken via an exam contract

Teaching methods

Seminar, Lecture

Extra information on the teaching methods

Ufora will be used.
The learning material will be provided in English.

Study material

Type: Syllabus

Name: Lecture notes on Model Reduction and Multiscale Numerical Methods
Indicative price: Free or paid by faculty
Optional: no
Language : English
Number of Pages : 200
Available on Ufora : Yes
Online Available : No
Available in the Library : No

Available through Student Association : No

References

lecture notes will be provided

Course content-related study coaching

The students can make appointments with the lecturer to have extra individual coaching.

Assessment moments

end-of-term and continuous assessment

Examination methods in case of periodic assessment during the first examination period

Presentation, Written assessment

Examination methods in case of periodic assessment during the second examination period

Presentation, Written assessment

Examination methods in case of permanent assessment

Assignment

Possibilities of retake in case of permanent assessment

examination during the second examination period is possible

Extra information on the examination methods

End-of-term evaluation: Written assessment (project report) and presentation
Permanent evaluation: (bi-)weekly homework assignments

Calculation of the examination mark

Continuous (30%) and end-of-term assessment (70%)