GHENT UNIVERSITY

Course Specifications

Valid in the academic year 2022-2023

Vector and Function Spaces (C004213)

Course size *(nominal values; actual values may depend on programme)*

Credits 5.0		**Study time 150 h**		**Contact hrs**	45.0h

Course offerings and teaching methods in academic year 2022-2023

A (semester 1)	Dutch	Gent	lecture	30.0h
			seminar: coached exercises	15.0h

Lecturers in academic year 2022-2023

Haegeman, Jutho	WE05	lecturer-in-charge

Offered in the following programmes in 2022-2023	**crdts**	**offering**
Bachelor of Science in Physics and Astronomy	5	A

Teaching languages

Dutch

Keywords

vector, matrix, operator, matrix decompositions, eigenvalues, singular values, tensor, function space, Hilbert space, integral transform, Fourier transform, distribution, Euler-Lagrange equation, functional derivative, initial value problem, boundary value problem, Green's function

Position of the course

This course unit belongs to the learning pathway "Mathematics" in the Bachelor program Physics and Astronomy.
Linearity and linear equations arise throughout physics; consider Maxwell's equations or the Schrödinger equation. It is thus not surprising that techniques from linear algebra are often used to study these equations and construct solutions, both analytically and numerically. Indeed, techniques and algorithms from linear algebra are perfectly suited for large-scale computer implemtations and simulations.
This course has as its aim to introduce the student to the most important concepts and techniques from linear algebra, but also to illustrate how these concepts manifest themselves outside the narrow scope of finite-dimensional vectors and matrices, namely in function spaces and other infinite-dimensional vector spaces, which appear throughout physics.

Contents

1. Basic linear algebra
- mathematical structures
- vector spaces
- basis and dimensionality

2. Linear maps
- linear maps, operators and transformations
- linear functionals and dual spaces
- matrices and determinants, matrix product
- linear group and special linear group
- linear systems

3. Eigenvalue problems
- spectral decomposition, eigenvalues and eigenspaces
- Jordan normal form
- functions of linear operators

4. Inner product spaces

- normed vector spaces, Banach spaces, induced norms of linear maps
- inner product, Cauchy-Schwarz inequality
- orthogonality, Gram-Schmidt orthogonalisation, Hilbert spaces
- linear maps in Hilbert spaces, hermitian adjoint, unitary, hermitian and normal maps
- bilinear and quadratic forms; Sylvester's law of inertia

5. Unitary similarity and unitary equivalence

- unitary group
- elementary unitary transformations
- QR decomposition
- Schur decomposition
- singular value decomposition
- Krylov methods

6. Multilinear algebra

- tensor product of vector spaces and linear maps
- tensor product of Hilbert spaces
- tensor algebra and exterior (wedge) product

7. Function spaces

- norms in function spaces
- orthogonal polynomials: Legendre, Laguerre, Hermite, Tchebychef (application: numerical integration)
- operators in Hilbert spaces: integral and differential operators, boundedness, compactness, symmetric and self-adjoint operators
- spectral theory

8. Fourier analysis and distributions

- Fourier transform
- test functions and distributions: Dirac delta distrubition
- functionals, functional derivatives, Euler-Lagrange equation

9. Applications of differential operators

- adjoint of differential operators, boundary conditions
- initial value problems, fundamental solution
- boundary value problems, Green's functions
- Sturm-Liouville eigenvalue problems
- partial differential equations

Initial competences

Experience with vectors and matrices from Linear Algebra in the first bachelor year.

Final competences

1 Being able to detect and exploit linearity in physical applications beyond the scope of standard vectors and matrices.
2 Being able to use standard techniques in linear algebra (solving linear systems, suitable matrix decompositions, orthogonalisation) both analytically and computationally for analysing physical problems, and —at least equally important— knowing when to use which technique and when not, by knowing advantages and limitations of different techniques (e.g. assumptions on hermiticity, positive definiteness, required computational complexity, ...)
3 Being able to use standard tools in function spaces: Fourier and other integral transorms, Dirac-delta distribution, functional derivatives, orthogonal polynomials and other countable bases.
4 Being able to recognise initial and boundary value problems and know which techniques can be used to analyse them or to construct solutions

Conditions for credit contract

Access to this course unit via a credit contract is determined after successful competences assessment

Conditions for exam contract

This course unit cannot be taken via an exam contract

Teaching methods

Lecture, Seminar: coached exercises

Extra information on the teaching methods

Lectures in which the theory is presented. Exercise classes in which the students solve exercises under supervision, either with pen and paper, or using a computer.

Theory and exercises: because of COVID19, alternative didactic methods can be used when this is necessary.

Learning materials and price

Lecture notes will be distributed via Ufora in PDF format.

References

- "Mathematics for Physicists: Introductory Concepts and Methods", Alexander Altland and Jan Von Delft, Cambride University Press, 2019
- "A Physicist's Introduction to Algebraic Structures: Vector spaces, Groups, Topological Spaces, and more", Palsh B. Pal, Cambridge University Press, 2019
- "A Course in Modern Mathematical Physics: Groups, Hilbert Space and Differential Geometry", Peter Szekeres, Cambridge University Press, 2004
- "Mathematics for Physicists: An Illustrated Handbook", Adam Marsh, World Scientific, 2018
- "Manifolds, Tensors and Forms: An Introduction for Mathematicians and Phycisists", Paul Renteln, Cambridge University Press, 2014
- "Functional Analysis for Physics and Engineering: An Introduction", Hiroyuki Shima, CRC Press, 2015
- "Introduction to Hilbert Spaces with Applications", Lokenath Debnath and Piotr Mikusinski, Academic Press, 2005

Course content-related study coaching

Students can ask questions on the theory and the exercises before, during and after the lectures. This is also possible on appointment or by email. There is interactive support via the Ufora forum.

Assessment moments

end-of-term and continuous assessment

Examination methods in case of periodic assessment during the first examination period

Written examination with open questions

Examination methods in case of periodic assessment during the second examination period

Written examination with open questions

Examination methods in case of permanent assessment

Assignment

Possibilities of retake in case of permanent assessment

examination during the second examination period is possible

Extra information on the examination methods

Written exam in two parts: theory and exercises. The theory exam will test whether the student has learned the important concepts and know how they are related (without using the lecture notes). The exercise exam will test whether the student knows how to apply these concepts (using lecture notes).
Furthermore, there is a small scale project in which a somewhat larger exercise needs to be solved and reported upon, possibly using computer simulations.

Calculation of the examination mark

Theory part of the exam: 40%, exercise part: 40%, project: 20%.