GHENT UNIVERSITY

Course Specifications

Valid as from the academic year 2026-2027

Vector and Function Spaces (C004213)

Course size *(nominal values; actual values may depend on programme)*

Credits 5.0 **Study time 150 h**

Course offerings and teaching methods in academic year 2026-2027

A (semester 1)	Dutch	Gent	seminar
			lecture

Lecturers in academic year 2026-2027

Haegeman, Jutho	WE05	lecturer-in-charge

Offered in the following programmes in 2026-2027	crdts	offering
Bachelor of Science in Physics and Astronomy	5	A

Teaching languages

Dutch

Keywords

vector, matrix, operator, matrix decompositions, eigenvalues, singular values, tensor, function space, Hilbert space, integral transform, Fourier transform, distribution, Euler-Lagrange equation, functional derivative, initial value problem, boundary value problem, Green's function

Position of the course

This course unit belongs to the learning pathway "Mathematics" in the Bachelor program Physics and Astronomy.
Linearity and linear equations arise throughout physics; consider Maxwell's equations or the Schrödinger equation. It is thus not surprising that techniques from linear algebra are often used to study these equations and construct solutions, both analytically and numerically. Indeed, techniques and algorithms from linear algebra are perfectly suited for large-scale computer implemtations and simulations.
This course has as its aim to introduce the student to the most important concepts and techniques from linear algebra, but also to illustrate how these concepts manifest themselves outside the narrow scope of finite-dimensional vectors and matrices, namely in function spaces and other infinite-dimensional vector spaces, which appear throughout physics.

Contents

1. Basic linear algebra
- mathematical structures
- vector spaces
- linear independence, basis and dimensionality

2. Linear maps and matrices
- linear maps, operators and transformations
- kernel, image and rank-nullity theorem
- matrices and determinants
- general linear group and basis transforms
- linear functionals and dual spaces
- affine transformations
- linear maps in real and complex vector spaces
- systems of linear equations

3. Linear operators and eigenvalues
- powers and polynomials of operators

- spectral decomposition, eigenvalues and eigenspaces
- Jordan normal form
- functions of linear operators
- application: dynamical systems

4. Norm and distance

- normed vector spaces, Hölder norms
- Banach spaces
- norms for linear maps, induced norm, matrix norms
- applications: sensitivity of linear systems, Markov chains and stochastic matrices

5. Inner product and orthogonality

- bilinear and sesquilinear forms
- inner product, Cauchy-Schwarz inequality
- orthogonality and unitarity, Gram-Schmidt orthogonalisation, Hilbert spaces and orthonormal basis
- linear functionals and the duality of Hilbert space
- bounded linear maps in Hilbert spaces: adjoint, unitary, hermitian and normal maps
- application: least square solutions

6. Unitary similarity and unitary equivalence

- unitary group
- elementary unitary transformations: permutations, Givens rotations, Householder reflections, discrete Fourier transform
- QR decomposition
- Schur decomposition
- bilinear and quadratic forms revisited: Sylvester's law of inertia
- singular value and polar decomposition
- Krylov methods

7. Function spaces

- norms and inner products in function spaces
- orthogonal polynomials: Legendre, Laguerre, Hermite, Tchebychef (application: numerical integration)
- Fourier series: convergence criteria and Gibbs phenomenon
- operators in Hilbert spaces: integral and differential operators, boundedness, compactness, symmetric and self-adjoint operators
- introduction to the spectral theory of operators

8. Linear differential operators

- adjoint of differential operators, boundary conditions
- initial value problems, fundamental solution, method of Frobenius
- boundary value problems, Green's functions
- Sturm-Liouville eigenvalue problems
- partial differential equations

9. Fourier calculus and distributions

- Fourier transform
- test functions and distributions: Dirac delta distrubition, distributional limits and derivatives, Sokhotsky-Plemelj formula, Dirac comb distribution, Poisson formula
- Fourier analysis revisited: relating different Fourier transforms, sampling and Shannon- Nyquist theorem
- functionals, functional derivatives, Euler-Lagrange equation

10. Multilinear algebra

- tensor product of vector spaces and linear maps
- tensor product of Hilbert spaces
- tensor algebra and exterior (wedge) product

Initial competences

Experience with vectors and matrices from Linear Algebra in the first bachelor year.

Final competences

1 Being able to detect and exploit linearity in physical applications beyond the scope of standard vectors and matrices.

2 Being able to use standard techniques in linear algebra (solving linear systems, suitable matrix decompositions, orthogonalisation) both analytically and computationally for analysing physical problems, and —at least equally important— knowing when to use which technique and when not, by knowing

advantages and limitations of different techniques (e.g. assumptions on hermiticity, positive definiteness, required computational complexity, ...)

3 Being able to use standard tools in function spaces: Fourier and other integral transorms, Dirac-delta distribution, functional derivatives, orthogonal polynomials and other countable bases.

4 Being able to recognise initial and boundary value problems and know which techniques can be used to analyse them or to construct solutions

Conditions for credit contract

Access to this course unit via a credit contract is determined after successful competences assessment

Conditions for exam contract

This course unit cannot be taken via an exam contract

Teaching methods

Seminar, Lecture, Independent work

Extra information on the teaching methods

Lectures in which the theory is presented. Exercise classes in which the students solve exercises under supervision, either with pen and paper, or using a computer.

Study material

Type: Syllabus

Name: Vector and Function Spaces
Indicative price: Free or paid by faculty
Optional: no
Language : English
Number of Pages : 384
Available on Ufora : Yes
Online Available : Yes
Available in the Library : No
Available through Student Association : No

Type: Slides

Name: Vector and Function Spaces
Indicative price: Free or paid by faculty
Optional: yes
Language : Dutch
Available on Ufora : Yes
Online Available : Yes
Available in the Library : No
Available through Student Association : No

Type: Handouts

Name: Exercise bundle and solutions
Indicative price: Free or paid by faculty
Optional: yes
Language : English
Number of Pages : 60
Available on Ufora : Yes
Online Available : No
Available in the Library : No
Available through Student Association : No
Usability and Lifetime within the Course Unit : intensive
Usability and Lifetime within the Study Programme : one-time
Usability and Lifetime after the Study Programme : not

References

- "Mathematics for Physicists: Introductory Concepts and Methods", Alexander Altland and Jan Von Delft, Cambride University Press, 2019
- "A Physicist's Introduction to Algebraic Structures: Vector spaces, Groups, Topological Spaces, and more", Palsh B. Pal, Cambridge University Press, 2019
- "A Course in Modern Mathematical Physics: Groups, Hilbert Space and Differential Geometry", Peter Szekeres, Cambridge University Press, 2004
- "Mathematics for Physicists: An Illustrated Handbook", Adam Marsh, World Scientific, 2018

- "Manifolds, Tensors and Forms: An Introduction for Mathematicians and Phycisists", Paul Renteln, Cambridge University Press, 2014
- "Functional Analysis for Physics and Engineering: An Introduction", Hiroyuki Shima, CRC Press, 2015
- "Introduction to Hilbert Spaces with Applications", Lokenath Debnath and Piotr Mikusinski, Academic Press, 2005

Course content-related study coaching

Students can ask questions on the theory and the exercises before, during and after the lectures. This is also possible on appointment or by email. There is interactive support via the Ufora forum.

Assessment moments

end-of-term and continuous assessment

Examination methods in case of periodic assessment during the first examination period

Written assessment with open-ended questions

Examination methods in case of periodic assessment during the second examination period

Written assessment with open-ended questions

Examination methods in case of permanent assessment

Assignment

Possibilities of retake in case of permanent assessment

examination during the second examination period is possible

Extra information on the examination methods

Written exam in two parts: theory and exercises. The theory exam will test whether the student has learned the important concepts and know how they are related (using lecture notes). The exercise exam will test whether the student knows how to apply these concepts (using lecture notes).
Throughout the semester, a larger exercise/project needs to be solved.

Calculation of the examination mark

The final score is the weighted average of the scores for the permanent assessment (exercise/project: 10%) and for the periodic assessment (theory exam: 45%, exercise exam: 45%).