GHENT UNIVERSITY

Course Specifications

Valid in the academic year 2023-2024

Combinatorial Algebraic Geometry (C004398)

Course size *(nominal values; actual values may depend on programme)*

Credits 6.0 **Study time 180 h**

Course offerings in academic year 2023-2024

A (semester 1)	English	Gent

Lecturers in academic year 2023-2024

Mohammadi, Fatemeh	WE01	lecturer-in-charge

Offered in the following programmes in 2023-2024	**crdts**	**offering**

Teaching languages

English

Keywords

Toric varieties/ideals, polyhedral geometry, Triangulations of polytope, Ehrhart polynomial

Position of the course

This course is an introductory course to Combinatorial Algebraic Geometry. It is suitable for MSc students, Ph.D. students, and motivated Undergraduate students with basic knowledge in Algebra.

Contents

Combinatorial algebraic geometry is the study of varieties with a combinatorial structure. Toric varieties form an important part of this field. Due to combinatorial tools, toric varieties are well-studied and they play an important role in commutative algebra, algebraic geometry and combinatorics. This lecture series will provide an introduction to toric varieties. We will focus more on examples and methods to generate such varieties. There are many great textbooks and lecture notes on toric geometry. I will only present some selected topics on toric geometry following Bernd Sturmfels' book on Gröbner bases and polytopes, and Cox's lecture notes on toric geometry. Here is a rough plan:
Course organization and scope: The list of main contents to cover are

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1 The theory of Gröbner bases and Gröbner fan
2 A crash course in polyhedral geometry
3 Toric ideals
4 Triangulations of polytopes
5 Ehrhart polynomial
6 Lattice ideals
7 Toric degenerations of varieties
8 Grassmannians
The lectures will not be recorded and the students cannot take the course if they are unable to attend it.

Initial competences

Some familiarity with the basic elements of algebra 1 and 2 (e.g., fields, groups, rings, ideals and modules) is required.

Final competences

1 The student knows the fundamental objects in combinatorial algebraic geometry

and can solve advanced problems concerning lattice polytopes, their triangulations, their algebraic counterparts, namely toric varieties.

2 The student is familiar with these concepts and has a good overview of these topics. The homeworks are designed to lead to potential research projects.

Conditions for credit contract

Access to this course unit via a credit contract is determined after successful competences assessment

Conditions for exam contract

This course unit cannot be taken via an exam contract

Teaching methods

Seminar, Lecture, Independent work

Extra information on the teaching methods

Eight homeworks (one for each subtopic) will be assigned based on audience background and their interests. Collaboration is both allowed and encouraged, but everyone must write up the solution by theirselves

Learning materials and price

Lecture notes will be available gratis.

References

- D. A. Cox, J. Little, and D. O'Shea: Ideals, Varieties, and Algorithms: an introduction to computational algebraic geometry and commutative algebra
- David Cox: Lectures on toric varieties
- R. Stanley: Combinatorics and commutative algebra
- Bernd Sturmfels: Gröbner bases and convex polytopes

Course content-related study coaching

Students can contact the lecturer or an assistant for questions during office hours. They will be specific times allocated for questions/discussion.

Assessment moments

continuous assessment

Examination methods in case of periodic assessment during the first examination period

Examination methods in case of periodic assessment during the second examination period

Examination methods in case of permanent assessment

Participation, Assignment

Possibilities of retake in case of permanent assessment

examination during the second examination period is possible in modified form

Extra information on the examination methods

The evaluation of this course will be based on weekly homework. A number of homeworks (one for each week/lecture) will be assigned based on audience background and their interests. Collaboration is both allowed and encouraged, but everyone must write up the solution and submit by theirselves. Homeworks will be posted here after each lecture and they are due (by email) a week after the assignment. Each homework will be graded and written feedback will be provided to students

Calculation of the examination mark

Each homework counts evenly for the total.