



stochastically) on the properties and the design of optimal controllers. Teach the students how to address and solve concrete and practical problems in this field.

## Contents

### TOPICS:

- Models for linear systems: Input-output model, State representations, Causality, stationarity and linearity, Impulse response matrix of a linear system, Transfer matrix of a linear stationary system, Solving the state equations for linear systems, Equilibria of stationary systems and their stability
- Stochastic signals: Definition of a stochastic signal, Probabilistic characteristics of a stochastic signal, Relations between stochastic input and output signals
- Controllability and pole placement: Notions of controllability and their definitions, Controllability criteria, The Kalman controllability decomposition, State feedback and pole placement
- Observability and state estimation: Notions of observability and their definitions, Observability criteria, The duality principle, The Kalman observability decomposition, State estimation, The separation principle
- Optimal control with a quadratic cost: Optimisation for a finite time interval, Optimisation for an infinite time interval, Dealing with deterministic perturbations, Dealing with stochastic perturbations
- Optimal state estimation: Optimal and optimal linear estimators, The Kalman-Bucy filter, The stationary Kalman-Bucy filter
- Optimal control using output measurements: Formulae for the optimal controller, Separation theorems

NOTIONS: State model, input-output model, linearity, stationarity, stability; Stochastic signals and their correlation functions and spectra; Controllability of a linear system; Kalman controllability decomposition; pole or eigenvalue placement; Observability of a linear system; Kalman observability decomposition; Realisability; Optimal control with a quadratic cost; Optimal state estimation, Kalman-Bucy filter; Combination of optimal control and optimal state estimation.

INSIGHTS: How to study the behaviour of a linear system?; How is a stochastic signal transformed by a linear system?; Can a linear stationary system be brought from one state to another; can it be stabilised by linear feedback?; Can its state be determined by observing its output?; Can a given input-output behaviour be realised by a state model?; How to design an optimal controller for linear systems with a quadratic cost function; what is the influence of perturbations in this control?; What is an optimal linear state estimator; how can it be designed?; How to perform an optimal control when only the outputs and not the states of a linear system are known?

## Initial competences

Linear algebra, systems of linear differential equations, probability theory

## Final competences

- 1 Knowing and understanding the consequences of the linearity and stationarity of a system; calculating the state trajectory of a linear stationary system.
- 2 Understanding when, and checking if a linear stationary system is controllable; stabilising it using linear state feedback.
- 3 Understanding when, and checking if a linear stationary system is observable; designing a Luenberger state observer/estimator.
- 4 Designing an optimal controller without and with input perturbations.
- 5 Understanding and working with expectations, covariance matrices, and optimal linear estimators.
- 6 Designing a Kalman-Bucy filter under output noise.
- 7 Designing the optimal combination of optimal controller and optimal state estimator.

## Conditions for credit contract

Access to this course unit via a credit contract is determined after successful competences assessment

## Conditions for exam contract

This course unit cannot be taken via an exam contract

## Teaching methods

Guided self-study, Lecture, Seminar: coached exercises

**Learning materials and price**

Lecture notes and additional course material through the electronic learning platform (freely downloadable).

**References**

- Linear systems, Thomas Kailath, Prentice-Hall, 1980.

**Course content-related study coaching****Assessment moments**

end-of-term assessment

**Examination methods in case of periodic assessment during the first examination period**

Written examination with open questions

**Examination methods in case of periodic assessment during the second examination period**

Written examination with open questions

**Examination methods in case of permanent assessment****Possibilities of retake in case of permanent assessment**

not applicable

**Extra information on the examination methods**

During examination period: written closed-book exam. Second chance: written closed-book exam.

**Calculation of the examination mark**

The exam takes 100% of the score. It consists of, on the one hand, theoretical questions and theoretical exercises testing for insight, and on the other, exercises in the style of the exercises that are offered, solved and discussed in the seminars.